Discussion Problems 6

Problem One: Undecidable Languages

Using self-referential TMs, prove that each of the following languages are undecidable.

i. $L = \{ \langle M \rangle \mid M \text{ is a TM that accepts a finite, even number of strings} \}$

ii. $L = \{ \langle M, w_1, w_2 \rangle \mid M \text{ is a TM, } w_1 \text{ and } w_2 \text{ are strings, and } M \text{ accepts } w_1 \text{ and } w_2 \}$

Problem Two: Non-RE Languages

Using self-referential TMs, prove that each of the following languages is unrecognizable:

i. $L = \{ \langle M \rangle \mid M \text{ is a TM that accepts a finite, even number of strings} \}$

ii. $L = \{ \langle M \rangle \mid M \text{ loops on at least one string} \}$

Problem Three: Non-co-RE Languages

Using self-referential TMs, prove that each of the following languages is co-unrecognizable (that is, these languages are not in co-**RE**). To do so, prove that the complement of each language is not an **RE** language.

i. $L = \{ \langle M, w_1, w_2 \rangle \mid M \text{ is a TM, } w_1 \text{ and } w_2 \text{ are strings, and } M \text{ accepts } w_1 \text{ and } w_2 \}$

ii. $L = \{ \langle M \rangle \mid M \text{ is a TM that accepts a finite, even number of strings} \}$

Problem Four: NTMs and Verifiers

i. Show that $L = \{ \langle M, w_1, w_2 \rangle \mid M \text{ is a TM, } w_1 \text{ and } w_2 \text{ are strings, and } M \text{ accepts at least one of } w_1 \text{ and } w_2 \}$ is an **RE** language by defining a verifier for it. Briefly justify why your machine is a verifier for L.

ii. Show that $L = \{ \langle M, w_1, w_2 \rangle \mid M \text{ is a TM, } w_1 \text{ and } w_2 \text{ are strings, and } M \text{ accepts at least one of } w_1 \text{ and } w_2 \}$ is an **RE** language by defining an NTM for it. Briefly justify why your machine is an NTM for L.